九十六學年第一學期 PHYS2310 電磁學 期末考試題(共兩頁)

[Griffiths Ch. 5-6] 2008/01/08, 10:10am–12:00am, 教師：張存續

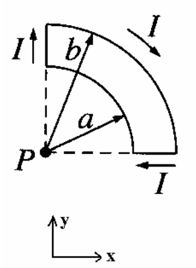
記得寫上學號，班別及姓名等。請依題號順序每頁答一題。

◇ Useful formulas: Cylindrical coordinate $\nabla\times\mathbf{v}=[\frac{1}{s}\frac{\partial v_z}{\partial\phi}-\frac{\partial v_\phi}{\partial z}]\hat{\mathbf{s}}+[\frac{\partial v_s}{\partial z}-\frac{\partial v_z}{\partial s}]\hat{\phi}+\frac{1}{s}[\frac{\partial(sv_\phi)}{\partial s}-\frac{\partial v_s}{\partial\phi}]\hat{\mathbf{z}}$

◇ Specify the magnitude and direction for a vector field.

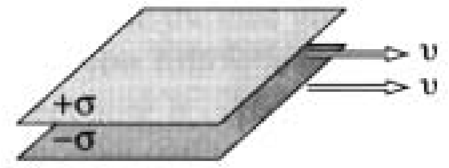
1. (8%,6%,6%) Explain the following terms as clear as possible.
   (a) Paramagnetism, diamagnetism, and ferromagnetism.
   (b) Hysteresis (draw a hysteresis loop).
   (c) Curie temperature.

2. (10%, 10%) A steady current loop is placed in a uniform magnetic field as shown in the figure. The uniform magnetic field is $B_0\hat{\mathbf{z}}$.
   (a) Find the magnetic field **B** at point *P* generated by the loop.
   (b) Find the force **F** on the loop.

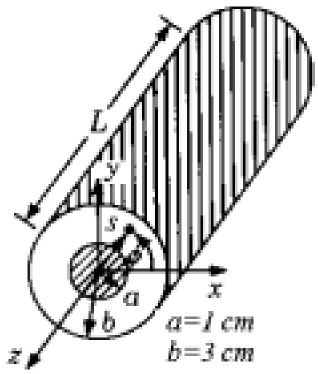


3. (10%, 10%) Find the magnetostatic boundary conditions.
   (a) In terms of **B** and **K**.
   (b) In terms of **H**, **M** and $\mathbf{K}_f$.
   [Hint:
   1. Write the equations of divergence **B/H** and use Gauss's law to find *normal* conditions.
   2. Write the equations of curl **B/H** and use Ampere's law to find *tangential* conditions.]

4. (7%, 7%, 6%) A large parallel-plate capacitor, with uniform surface charge σ on the upper plate and $-\sigma$ on the lower, is moving with a constant speed v, as shown in the figure.
   (a) Find the magnetic field between the plates and also above and below them.
   (b) Find the magnetic force per unit area on the lower plate (attractive or repulsive force).
   (c) At what speed v would the magnetic force balance the electric force?



5. (7%, 7%, 6%) A coaxial line of length L with inner and outer conductor radii of 1 cm and 3 cm, respectively, is filled with a ferromagnetic material. When the material is subjected to a magnetic field, $\mathbf{H}(s,\phi,z) = 1/s\,\hat{\phi}$ (A/m), it induces a magnetization, $\mathbf{M}(s,\phi,z) = 600/s\,\hat{\phi}$ (A/m). Determine
   (a) The bounded volume current density within the material.
   (b) The bounded surface current density on inner and outer surfaces.
   (c) The *relative* permeability of the material μ_r.
   [Hint: $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{N/A^2}$ and $\mathbf{B} = \mu_0 \mu_r \mathbf{H}$].



1. Textbook Chs.5 and6

(a) When a magnetic field is applied, a net alignment of these magnetic dipoles occurs, and the medium becomes magnetically polarized, or magnetized.

Paramagnetism: The magnetic polarization **M** is parallel to **B.**

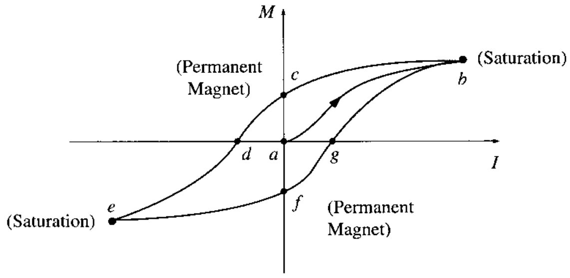
Diamagnetism: The magnetic polarization **M** is opposite to **B**.

Ferromagnetism: Substances retain their magnetization even after the external field has been removed.

(b) *Hysteresis*: Substances retain their magnetization even after the external field has been removed.

In the experiment, we adjust the current *I*, i.e. control **H**.

In practice *M* is huge compared to *H*.



(c) *Curie temperature*: As the temperature increases, the alignment is gradually destroyed. At certain temperature the iron completely turns into paramagnet. This temperature is called the curie temperature.

2. Problems 5.9 + 5.10

(a) The straight segments produce no field at *P*.

The two quarter-circles gives: $\frac{\mu_0 I}{8}(\frac{1}{a}-\frac{1}{b})\hat{\mathbf{z}}$

(b)

$$\begin{aligned}\mathbf{F}_{\text{mag}} &= -I\int(\mathbf{B}\times d\mathbf{l}) = IB_0\left[(b-a)\hat{\mathbf{x}}+\int_{\pi/2}^{0} b(\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}})d\theta+(b-a)\hat{\mathbf{y}}+\int_0^{\pi/2} a(\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}})d\theta\right]\\ &= IB_0\left[(b-a)\hat{\mathbf{x}}-\int_0^{\pi/2} b(\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}})d+(b-a)\hat{\mathbf{y}}+\int_0^{\pi/2} a(\cos\theta\hat{\mathbf{x}}+\sin\theta\hat{\mathbf{y}})d\theta\theta\right]\\ &= IB_0\left[(b-a)\hat{\mathbf{x}}+(-b+a)\hat{\mathbf{x}}+(b-a)\hat{\mathbf{y}}+(-b+a)\hat{\mathbf{y}}\right]\\ &= 0\end{aligned}$$

3. Textbook Chs.5 and6

(a)

Normal: $\nabla\cdot\mathbf{B}=0$. Consider a wafer-thin pillbox. Gauss's law states that $\oint_S \mathbf{B}\cdot d\mathbf{a}=0$.

The sides of the pillbox contribute nothing to the flux, in the limit as the thickness ε goes to zero.

$(B_{\text{above}}^{\perp}-B_{\text{below}}^{\perp})A=0 \quad\Rightarrow\quad B_{\text{above}}^{\perp}=B_{\text{below}}^{\perp}$.

Tangential: $\nabla\times\mathbf{B}=\mu_0\mathbf{J}$. Consider a thin rectangular loop. The curl of the Ampere's law states that $\oint_P \mathbf{B}\cdot d\ell=\mu_0 I_{\text{enc}}$. The ends gives nothing (as ε→0), and the sides give

$(B_{\text{above}}^{//} - B_{\text{below}}^{//})\ell = \mu_0 K\ell \quad \Rightarrow \quad B_{\text{above}}^{//} - B_{\text{below}}^{//} = \mu_0 K$ or $\mathbf{B}_{above}^{//} - \mathbf{B}_{below}^{//} = \mu_0(\mathbf{K}\times\hat{\mathbf{n}})$

(b)

Normal: $\nabla\cdot\mathbf{H} = -\nabla\cdot\mathbf{M}$. Consider a wafer-thin pillbox. Gauss's law states that $\oint_S \mathbf{H}\cdot d\mathbf{a} = -\oint_S \mathbf{M}\cdot d\mathbf{a}$. The sides of the pillbox contribute nothing to the flux, in the limit as the thickness ε goes to zero. $H_{above}^{\perp} - H_{below}^{\perp} = -(M_{above}^{\perp} - M_{below}^{\perp})$.

Tangential: $\nabla\times\mathbf{H} = \mathbf{J}_f$. Consider a thin rectangular loop. The curl of the Ampere's law states that $\oint_P \mathbf{H}\cdot d\ell = \mu_0 I_{f\,\text{enc}}$. The ends gives nothing (as ε→0), and the sides give

$(H_{\text{above}}^{//} - H_{\text{below}}^{//})\ell = \mu_0 K_f \ell \quad \Rightarrow \quad H_{\text{above}}^{//} - H_{\text{below}}^{//} = \mu_0 K_f$ or $\mathbf{H}_{above}^{//} - \mathbf{H}_{below}^{//} = \mathbf{K}_f \times\hat{\mathbf{n}}$.

4. Prob. 5.16

(a) According to the boundary conditions, the top plate produces a parallel field $\mu_0 K/2$, pointing out of the page for points above it and into the page for points below) The bottom plate produces a parallel field $\mu_0 K/2$, pointing into the page for points above it and out of the page for points below). Between the plates, the fields add up to $B = \mu_0 K = \mu_0 \sigma v$.

Above and below both plates, the fields cancel $B = 0$.

(b) $d\mathbf{F} = \mathbf{I}d\ell\times\mathbf{B} = \mathbf{K}da\times\mathbf{B} = \mathbf{J}d\tau\times\mathbf{B}$

$d\mathbf{F} = \mathbf{K}da\times\mathbf{B} \Rightarrow dF = \mathbf{K}\times\mathbf{B}da$

$\frac{dF}{da} = \mathbf{K}\times\mathbf{B} = \sigma v\frac{\mu_0\sigma v}{2} = \frac{\mu_0\sigma^2 v^2}{2}$ (repulsive force per unit area)

(c) The electric force of the plates is attractive $\frac{dF_E}{da} = \sigma\mathbf{E} = \sigma\frac{\sigma}{\varepsilon_0} = \frac{\sigma^2}{\varepsilon_0}$ (attractive force per unit area)

Balance: $\frac{dF}{da} = \frac{d(F_B + F_E)}{da} = \frac{\mu_0\sigma^2 v^2}{2} - \frac{\sigma^2}{2\varepsilon_0} = 0 \quad \Rightarrow \quad v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$ the speed of light.

5.

(a) $\mathbf{M}(s,\phi,z) = 600/s\,\hat{\phi}$ (A/m), $\mathbf{J}_b = \nabla\times\mathbf{M}$

$$\nabla\times\mathbf{M} = [-\frac{\partial M_\phi}{\partial z}]\hat{\mathbf{s}} + \frac{1}{s}[\frac{\partial(sM_\phi)}{\partial s}]\hat{\mathbf{z}} = 0\hat{\mathbf{s}} + \frac{1}{s}[\frac{\partial(600)}{\partial s}]\hat{\mathbf{z}} = 0 \Rightarrow \mathbf{J}_b = 0$$

(b) $\mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}} = M\hat{\phi}\times\hat{\mathbf{n}}$

(1) inner (s =1 cm, $\hat{\mathbf{n}} = -\hat{\mathbf{s}}$): $\mathbf{K}_b(s = 1\text{ cm}) = M\hat{\phi}\times(-\hat{\mathbf{s}}) = \frac{600}{0.01} = 60000\hat{\mathbf{z}}$ (A/m)

(2) outer (s =3 cm, $\hat{\mathbf{n}} = \hat{\mathbf{s}}$): $\mathbf{K}_b(s = 3\text{ cm}) = M\hat{\phi}\times(\hat{\mathbf{s}}) = -\frac{600}{0.03} = -20000\hat{\mathbf{z}}$ (A/m)

(c) $\mathbf{M} = \chi_m\mathbf{H} \Rightarrow \chi_m = 600/1 = 600$

$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mu_0(1+\chi_m)\mathbf{H} = \mu_0\mu_r\mathbf{H} \quad \Rightarrow \quad \mu_r = 601$